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Approximations of The Prime Soft Ideal and Maximal Soft Ideal of soft semirings

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Abstract

The aim of this paper is study the concepts of approximations (upper and lower) of ideal on the soft semirings. Moreover, we introduce the rough prime soft ideal and maximal soft Ideal. However, we study some of the properties of these approximations.
Keywords: Upper Approximation; Lower Approximation; Semiring; Softsemirin; Soft ideal

Introduction

The rough set theory has shown by Pawlak [1] in 1982. It was coming after a long term in information system and proposed as good formal tool for modeling and processing incomplete information in information system. In recently 40 years many researches develop this theory and use it in many areas. It is coming as an extension of the set theory, in which a subset of a universe is described by lower and upper approximations. The upper approximation of a given set is the union of all the equivalence classes which are subsets of the set, and the upper approximation is the union of all the equivalence classes which are intersection with set non-empty.

Many researchers develop this theory and use rough theory in algebra. For example, the notation of rough subring with respect ideal has presented by B.Davvaz [2]. Algebraic properties of rough sets have been studied by Bonikowaski [3], and Pomykala [4]. John N. Mordeson [5], he used covers of the universal set to defined approximation operators on the power set of the given set. Some concept lattice in rough set theory has studied by YY Yao [6]. Ronnason Chinram, [7], his study Rough prime ideas and Rough fuzzy prime ideals in Gamma-semigroups. The lattice theoretical approach has been showed that the set of rough sets for-ms a Stone algebra by Iwinski [8]. Comer [9] discuss the rough sets and various algebra, his study algebraic logic, such as Stone algebras and relation algebras. Some other substitute an algebraic structure instead of the universe set. Like Biswas and Nanda [10], they make some notions of rough subgroups. Kuroki in [11], introduced the notion of a rough ideal in a semi group. Some properties of the Also, Kuroki and Mordeson in [12] studied the structure of rough sets and rough groups. In addition, B.Davvaz [13] applied the concept of approximation spaces in the theory of algebraic hyperstructures, and in investigated the similarity between rough membership functions and condi-tional probability. And S.B Hosseinin[14], he introduced and discussed the concept of T-rough (prime, primary) ideal and T-rough fuzzy (prime, primary) ideal in a commutative ring .[15] introduce the notion of rough ideals of a semiring with respect to the Bourne relation induced by an ideal of a semiring Rough ideals in Semirings. In this paper, we introduce the notion of roughness of an ideal of a soft semiring with respect to the equivalence relation induced by an ideal of a soft semiring. In addition, we give some properties of such ideals.

Preliminaries

Definition 1: Suppose that U (*universe*) be a nonempty finite set. Let R *an equivalence relation* (reflexive, symmetric, and transitive) on an U. For an approximation space (U, R) ,we define the upper approximation of X by$\overline{RX} = \{x \in U : [x]_R \cap X \neq \emptyset\}$,i.e. $\overline{RX}$ the set of all objects which can be only classified as *possible* members of X with respect to R is called the *R-upper approximation* of a set X with respect to R. And the lower approximation of X by $\underline{RX} = \{x \in U : [x]_R \subseteq X\}$, i.e $\underline{RX}$ is the set of all objects which can be with *certainty* classified as members of X with respect to R is called the *R-lower approximation* of a set X with respect to R. We use U/R to denote the family of all equivalent classes of R and $[\boldsymbol{x}]_R$ to denote an equivalence class in R containing an element $x \in U$. The empty set ϕ and the element of U/R called elementary set. For any $X \subseteq U$, we write X^c to denote the complementation of X in U.

Definition 2: For an approximation space (U, R), we define the *boundary region* by $BX_R = \overline{RX} - \underline{RX}$, i.e. BX_R is the set of all objects which can be decisively classified neither as members of X nor as members of X^c with respect to R. if $BX_R = \emptyset$, we say X is exact (*crisp)* set. But if $BX_R \neq \emptyset$, we say X *Rough set* (inexact).

Definition 3: We say S is semiring if S *is* a nonempty set with two binary operations on S called $(+)$ addition and $(*)$ multiplication such that:
• $(S,+)$ is a semigroup,
• $(S,*)$ is a semigroup, and
• $x*(y + z) = x*y+x*z$ and $(x + y)*z = x*z + y*z$ for all $x,y,z \in S$.
• if $x+y=y+x$ for all $x,y \in S$, then we called S is additively commutative.
Note that, for all $x \in$ S. such that $0 \cdot x = x \cdot 0 = 0$ and $0 + x = x + 0 = x$, we called 0 is a zero element of a semiring S.

Definition 4: A nonempty subset I of a semiring S is called a left [right] ideal of S if I is closed under addition and $SI \subseteq I$ [$IS \subseteq I$, respectively]. We say that I is an ideal of S, denoted by I ◄ S, if it is both a left and a right ideal of the semiring S.

Definition 5: Suppose that U be an initial universe set and E be a set of parameters. The power set of U is denoted by $P(U)$ and A is a subset of E. A pair (η, A) is called a soft set over U, where η is a mapping given by $\eta: A \to P(U)$. The set Supp $(\eta, A) = \{x \in A \mid \eta(x) \neq \emptyset\}$ is called the support of the soft set (η, A). Thus a null soft set in [18] is indeed a soft set with an empty support, and we say that a soft set (η, A) is non-null if Supp $(\eta, A) \neq \emptyset$.

Definition 6: Suppose that (η, A) is a non-null soft set over a semiring S. Then (η, A) is called a *soft semiring* over S if $\eta(x)$is a subsemiring of S for all $x \in$Supp (η, A).

Definition 7: Suppose that (α, A) and (β, B) is two soft semirings over S. Then the soft semiring (β, B) is called a soft subsemiring of (α, A) if it satisfies:

(1) $B \subset A$;

(2) $\beta(x)$ is a subsemiring of $\alpha(x)$ for all $x \in$ Supp (β, B).

Definition 8: Suppose that (η, A) is a soft semiring over a semiring S. A non-null soft set (γ, I) over S is called *a soft ideal* of (η, A), denoted by (γ, I)e◄(η, A), if it satisfies:

(i) $I \subset A$;

(ii) $\gamma(x)$ is an ideal of $\eta(x)$ for all $x \in$ Supp (γ, I).

Remark 1: Every soft ideal of a soft semiring (η, A) over S is a soft subsemiring of (η, A), but not every soft subsemiring of (η, A) is a soft ideal.

Example 1: Suppose that $S=Z_6 = \{0, 1, 2, 3, 4, 5\}$ is the semiring of integers module 6. Suppose $(A=Z_6) \subseteq S$ and (η, A) is a soft set over Z_6, where $\eta: A \to P(Z_6)$ is a set-valued function defined by $\eta(x) = \{y \in Z6 \mid x \rho y \Leftrightarrow xy \in \{0, 2, 4\}\}$ for all $x \in A$.

Then we have $\eta(0)=\eta(2)=\eta(4)=Z_6$ and $\eta(1)=\eta(3)=\eta(5)=\{0, 2, 4\}$ are subsemirings of Z_6. Hence (η, A) is a soft semiring over Z_6.

Example 2: Consider example 2.1. Suppose that (γ, I) is a soft set over Z_6, where $I = \{0, 1, 2\}$ and $\gamma: I \to P(Z_6)$ is a set-valued function defined by $\gamma(x) = \{y \in Z_6 \mid x \rho y \Leftrightarrow xy = 0\}$ for all $x \in I$. Then $\gamma(0) = Z_6$◄ $Z_6 = \eta(0)$, $\gamma(1) = \{0\}$◄$\{0, 2, 4\} = \eta(1)$, and $\gamma(2) = \{0, 3\}$ ◄$Z_6 = \eta(2)$. Hence (γ, I) is a soft ideal of (η, Z_6).

Example 3: in example 2.1. If suppose that (α, I) be a soft set over Z_6, where $I = \{2, 3, 4, 5\}$ and $\alpha: I \to P(Z_6)$ is a set-valued function defined by $\alpha(x) = \{y \in Z_6 \mid x\rho y \Leftrightarrow xy = 0\}$ for all $x \in I$. Then $\alpha(2) = \{0, 3\}$◄$Z_6 = {}_{\eta}(2)$, $\alpha(3) = \{0, 2, 4\}$◄$\{0, 2, 4\} = \eta(3)$, $\alpha(4) = \{0, 3\}$◄$Z_6 = \eta(4)$, and $\alpha(5) = \{0\}$◄$\{0, 2, 4\} = \eta(5)$, and so (α, I) is a soft ideal of (η, A).

Definition 7: *Suppose that* (η, A) *is a soft semiring.* We defines an equivalence relation $\sim_I$ on η by $r \sim_I$ if and only if there exists elements a and a' of a ideal I of a soft semiring (η, A) satisfying $(r+a)_{\sim=} r'+a'$. Note that if $r_{\sim I}\, r'$ and $s_{\sim I}\, s'$ then $(r+s)_{\sim I}(r'+s')$ and $(r*s)_{\sim I}(r'*s')$.

Approximations of Soft semirings

Throughout this paper we use the approximation space $(\eta, \sim_I)$ where η is a soft semiring and $\sim_I$ is the relation induced by the ideal I of η. We use the notation $[\eta]_I$ to denote the set of all equivalence classes of elements of R under this relation and the equivalence class of an element r of η by $[r]_I$.

Definition 1: The lower and upper approximation of a subset X of η in this approximation space $(\eta,\sim_I)$ are respectively given by $\underline{\eta X}$ = {x∈ η: $[x]_I \subseteq X$}and $\overline{\eta X}$ = {x ∈ η: $[x]_I \cap X \neq \emptyset$}. i.e if $\underline{\eta X} \neq \overline{\eta X}$

(i) X is called a lower (upper) soft rough ring, if $\underline{\eta X}$and $\overline{\eta X}$ *are* a subring of R;

(ii) X is called a soft rough ring w.r.t. S of R, if $\underline{\eta X}$ and $\overline{\eta X}$ are subrings of R.

Proposition 1: For the approximation space $(\eta,\sim_I)$ and every subsets A *and* B of η we have :

1) $\underline{\eta A} \subseteq A \subseteq \overline{\eta A}$.
2) $\underline{\eta \emptyset} = \overline{\eta \phi}$, $\underline{\eta U} = \overline{\eta U}$.
3) $\underline{\eta(A \cup A)} \supseteq \underline{\eta(A)} \cup \underline{\eta}(B)$.
4) $\underline{\eta(A \cap B)} = \underline{\eta(\eta)} \cap \overline{\eta(-B)}$.
5) $\overline{\eta(A \cup B)} = \overline{\eta(A)} \cup \overline{\eta(B)}$.
6) $\overline{\eta(A \cap B)} \subseteq \overline{\eta(A)} \cap \overline{\eta(B)}$.
7) $\underline{\eta A^C} = (\overline{\eta B})^c$.
8) $\overline{\eta A^C} = (\underline{\eta B})^c$
9) $\underline{\eta(\underline{\eta A})} = \overline{\eta(\underline{\eta A})} = \underline{\eta A}$.
10) $\overline{\overline{(\eta(\overline{\eta A})}} = \underline{\eta(\overline{\eta A})} = \overline{\eta A}$.

Example 1: Consider example 2.1. Let η =Z_6 = {0,1,2,3,4,5} be the soft semiring. Let I = {0, 1, 2} be a soft ideal of (η, Z_6). Suppose that $(A= Z_6) \subseteq \eta$. For x∈ η : $x + I$, we get {0,1,2}, {1,2,3 },{2,3,4 },{3,4,5 },{4,5,0 },{5,0,1 }. The upper approximations of A with respect of I is$\overline{I(\eta)}$ =∪ { $x \in \eta : (x + I) \cap A \neq \emptyset$}= {0, 1, 2, 3, 4, 5}.And the lower approximation of X with respect of I is $\underline{I(\eta)}$ =∪ { $x \in \eta : x + I \subseteq A$}={0, 1, 2, 3, 4, 5}. If we take I = {2, 3, 4, 5}, then for x∈ η : $x + I$, we get {2, 3, 4, 5}, {3, 4, 5, 0},{4, 5, 0, 1},{5, 0, 1, 2},{0, 1, 2, 3},{1, 2, 3, 4}. So,$\overline{I(\eta)}$ =∪ { $x \in \eta : (x + I) \cap A \neq \emptyset$}= {0, 1, 2, 3, 4, 5} and the lower approximation of A with respect of I is $\underline{I(\eta)}$ =∪ { $x \in \mathcal{R} : x + I \subseteq A$}={0, 1, 2, 3, 4, 5}.

Example 2: in example 3.1. Let η =Z_6 ={0,1,2,3,4,5} be a ring of integers modulo 6 and S = (F, A) be a soft set over R which is given by

	0	1	2	3	4	5
e_1	1	0	1	1	1	0
e_2	0	1	1	0	1	1

Then the mapping η: $R \rightarrow P(A)$ is given by η (0) = η (3) = {e_1}, η (1) = η (5) = {e_2}, η (2) = η (4) = {e_1, e_2}. Suppose that X = {0, 1, 2, 3} ⊆ η, we have $\underline{X(\eta)}$ = {0, 3} is subring and $\overline{X(\eta)}$ = Z_6 is subring. This shows that X is a soft rough ring of η.

Example 3: In example 3.1.Let Z_6 = {0, 1, 2, 3, 4, 5} be the ring of integers modulo 6 and ~ be a congruence on Z_6 with ρ-congruence classes $[0]_\sim$ = {0, 2, 4} and $[1]_\sim$ = {1, 3, 5}. Suppose that $A = Z_6$ and: $A \rightarrow P(\eta)$ is a set-valued function defined by $\eta(x) = \{y \in Z_6 | x{\sim}y \Leftrightarrow x + y \in \{0,1,2\}\}$for all $x \in A$. Then, η(0)={0,1,2}, $\underline{\eta(1)}$ = {0, 1, 5}, $\underline{\eta}$(2)={0,4,5}, η(3) ={3,4,5}, η(4)={2, 3, 4} and η(5) ={1, 2, 3}. Thus, $\overline{\eta(X)} = \sim(\overline{X(\eta)})$={$y$∈$Z_6$|$[y]_\sim \cap \underline{\eta(x)} = \emptyset$} =$Z_6$, for all $x \in A$, is a subring of Z6. Hence, is an upper rough soft ring over Z6. Not that if

η_1 *and* η_1 , are soft rings, then the soft intersection of $\eta_1$and η_2 is denoted by $\eta_1 \cap \eta_1$ and defined by $(\eta_1 \cap \eta_2)(x) = \eta_1(x) \cap \eta_2(x)$ for all $x \in A$.

Rough Prime Soft Ideal and Maximal Soft Ideal

In this section, we will introduce the notions of rough prime soft ideals and rough soft maximal ideals of a soft ring that introduce by M.R. Alimoradi in [19].

Definition 1: Assume that η is a soft ring over R. A soft ideal (φ, I) of η is called a prime soft ideal of, if φ(x) is a prime ideal of η(x) for all x in Supp(φ,I).
And is called a maximal soft ideal of η if φ(x) is a maximal ideal of η(x) for all x in Supp(φ,I).

Example 1: Suppose that η=Z_4 {0,1,2,3} and $I = \{0\}$. Suppose that: $A \rightarrow P(R)$ is the set-valued function defined via $\eta(x) = \{y \in \eta : xy \in \{0,2\}$. Since η (0)= $\eta(2) = Z_4$ and $\eta(1) = \eta(3) = \{0,2\}$, then η is a soft ring over R. Furthermore, by considering the function $\varphi : I \rightarrow P(\eta)$ given by $\varphi(x)=\{ y \in x + 2y = 0\}$, one has φ(0)={0,2} is a prime ideal of η (0) = Z_4 and so (φ, I) is a prime soft ideal of soft ring η. Also, (φ, I) is a maximal soft ideal of η, since φ(x) is a maximal ideal of $\eta(x)$ for all x in Supp (φ,I).

Example 1.1: Assume that A=η=Z and $I = \{1\}$ .Suppose that the set-valued functions $\eta : A \rightarrow$ p(η)given by $\eta(x) = \{nx : n \in Z\}$ and $\varphi(x) : I \rightarrow P(\eta)$ given by $\varphi(x) = \{n(x\text{-}1) : n \in Z\}$, one has η(x)=xZ is a subring of η for all $x \in$ A andφ(1)= {0{ is a prime ideal of $\eta(1) = Z$. Therefore η is a soft ring over R and (φ, I) is its prime soft ideal, but (φ, I) is not a maximal soft ideal.

Proposition 1: If J is a ideal of soft semiring, then $\overline{J(\eta)}$ is an ideal of η .
Proof. We assume that a, b ∈ $\overline{J(\eta)}$ and r ∈ η, then [a]I∩J ≠ ∅ and [b]J ∩ ≠ ∅. So, there exists x ∈[a]I∩J ≠ ∅ and y∈[b] /I ∩J ≠ ∅. Since J is an ideal of η , x +y ∈J and rx ∈ J. Now, x ∈[a]I and y ∈[b]I ⇒ x+y ∈ [a+b]I . Thus, [a+b]I∩J≠ ∅) , proving that a +b ∈$\overline{J(\eta)}$. Since x ∈[a]I, rx∈ [ra]I for any r ∈ η . Hence, [ra]I∩J≠ ∅. Therefore, ra ∈ $\overline{J(\eta)}$. In a similar way, we get $\underline{ar}$ ∈ $\overline{J(\eta)}$. Therefore, $\overline{J(\eta)}$ is an ideal of η.

Definition 2: If A and B are anon-empty subset of η, we denote AB for the set of all finite sums { $a_1 b_1 + a_2 b_2, \ldots, a_n b_n : n \in \mathbb{N}, a_i \in A, b_i \in B\}$. *i.e*: $AB = \sum_{i=1}^{n}(a_i \,.\, b_i)$, $a_i \in A$, $b_i \in B$.
Proposition 2: Suppose that I is an Ideal of η, and A, B are non-empty subset of the soft semiring η, then $\overline{I(A.B)} = \overline{I(A)}.\overline{I(B)}$■

Example 2: Let consider the soft ring $\mathbb{Z}_6$, I={0,1,2} and A={1,2,3,4,5 }, B={0,1,2,4},then $AB = \sum_{i=1}^{n}(a_i \,.\, b_i)$, $a_i \in A$, $b_i \in B$.
$\underline{AB}$= {0,1,2,3,4,5}. $\overline{I(A)} = \{0,1,2,3,4,5\}$. $\overline{I(B)} = \{0,1,2,3,4,5\}$.
$\overline{I(A.B)} = \{0,1,2,3,4,5\}$. $\overline{I(A)}.\overline{I(B)}$.= {0,1,2,3,4,,5}.

Proposition 3: Suppose that J is a soft ideal of soft semiring η and $\underline{J(\eta)} \neq \emptyset$, then $\underline{J(\eta)}$ is an ideal of η. *Proof.* Suppose $a,b \in \underline{J(\eta)}$ and $r \in \eta$. Then $[a]_I \subseteq J$ and $[b]_I \subseteq J$. then $[a+b]_I = [a]_b + [b]_I \subseteq J + J = J$. Therefore, $a+b \in \underline{J(\eta)}$. Now, $[ra]_I = [a]_I * [r]_I \subseteq J$, which proves that $ra \in \underline{J(\eta)}$. In similar way, $ar \in \underline{J(\eta)}$. Hence, $\underline{J(\eta)}$ is an ideal.

Conclusion

The notion of upper and lower approximation of soft semiring and we study these concepts of approximations (upper and lower) of ideal on the soft semirings. In addition, we introduce the concepts of Rough Prime Soft Ideal and Maximal Soft Ideal.

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